Ad Soyad:

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İST.257 İLERİ MATEMATİK FİNAL SINAVI SORULARI

1. Beta fonksiyonundan yaralanarak aşağıdaki integralleri hesaplayınız.

a) $\int_0^{\frac{\pi}{2}} \sin^3\theta\cos^2\theta d\theta = ?$ b) $\int_0^1 x^2(1-x)^3 dx = ?$

2. $f: IR^2 \to IR$, $f(x,y) = \dfrac{3x^2y}{x^4+y^2}$ fonksiyonunun $(x,y)\to(0,0)$ için limitini araştırınız.

3. $F(t) = 4t^3\vec{i} + 3\vec{j} + 2(t+1)\vec{k}$ ve $G(t) = \left(te^{-t^2}\right)\vec{i} + \left(\dfrac{1}{t\sqrt{\ln t}}\right)\vec{j} + (\tan t)\vec{k}$ fonksiyonlarının integrallerini bulunuz.

4. $\lim\limits_{t\to -2}\left(\dfrac{t^2-4}{t+2}\vec{i} + \ln(t+3)\vec{j} + \dfrac{\sin(t+2)}{t+2}\vec{k}\right) = ?$

5. $F(t) = e^t\vec{i} + \sqrt{1+t^2}\,\vec{j} + \ln t\,\vec{k}$ ve $G(t) = \sin t\,\vec{i} + \ln(1+t)\vec{j} + t\,\vec{k}$ fonksiyonları için $F\bullet G = ?$ ve $F\times G = ?$

BAŞARILAR

Süre: 90 dk.

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İLERİ MATEMATİK FİNAL SINAVI

CEVAP ANAHTARI

1) a) $\int_0^{\pi/2} \sin^3\theta \cos^2\theta \, d\theta = ?$

$$\int_0^{\pi/2} (\sin\theta)^{2x-1} (\cos\theta)^{2y-1} d\theta = \frac{1}{2} \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$$

$2x-1=3 \Rightarrow x=2$

$2y-1=2 \Rightarrow y=\frac{3}{2}$

$$\int_0^{\pi/2} \sin^3\theta \cos^2\theta \, d\theta = \frac{1}{2} \frac{\Gamma(2)\Gamma(\frac{3}{2})}{\Gamma(\frac{7}{2})}$$

$$= \frac{1}{2} \frac{1! \, \frac{1}{2}\cancel{\Gamma(\frac{1}{2})}}{\frac{5}{2}\,\frac{3}{2}\,\frac{1}{2}\cancel{\Gamma(\frac{1}{2})}} = \frac{2}{15}$$

b) $\int_0^1 x^2 (1-x)^3 dx = ?$

$$B(x,y) = \int_0^1 x^{t-1} (1-x)^{k-1} dx = \frac{\Gamma(t)\Gamma(k)}{\Gamma(t+k)}$$

$t-1=2 \quad t=3 \qquad k-1=3 \quad k=4$

$$= \frac{\Gamma(3)\Gamma(4)}{\Gamma(7)}$$

$$= \frac{2!\,3!}{6!} = \frac{1}{60}$$

2) $y=mx^2$ eğrileri boyunca yaklaşalım.

$$\lim_{(x,mx^2)\to(0,0)} \frac{3x^2(mx^2)}{x^4+(m^2x^4)} = \frac{3m}{1+m^2}$$

sonuç m ye göre değişeceğinden limit yoktur.

3) $\int F(t)\,dt = \left(\int 4t^3\,dt\right)\vec{i} + \left(\int 3dt\right)\vec{j} + \left(2\int (t+1)\,dt\right)\vec{k}$

$$= t^4\vec{i} + c_1 + 3t\vec{j} + c_2 + 2\left(\frac{t^2}{2}+t\right)k + c_3$$

$$= t^4\vec{i} + 3t\vec{j} + (t^2+2t)\vec{k} + c$$

$c = c_1 + c_2 + c_3$

$$\int G(t)\,dt = \left(\int t e^{-t^2}\,dt\right)\vec{i} + \left(\int \frac{1}{t\sqrt{\ln t}}\,dt\right)\vec{j} + \left(\int \tan t\,dt\right)\vec{k}$$

$$\int t e^{-t^2}\,dt = \frac{1}{2}\int e^{-u}\,du \qquad \left(t^2 = u \Rightarrow 2t\,dt = du\right)$$

$$= -\frac{1}{2}e^{-u} + c_1 = -\frac{1}{2}e^{-t^2} + c_1$$

$$\int \frac{1}{t\sqrt{\ln t}}\,dt = \int \frac{du}{\sqrt{u}} = 2\sqrt{u} + c_2 \qquad \ln t = u$$

$$\frac{1}{t}dt = du$$

$$= 2\sqrt{\ln t} + c_2$$

$$\int \tan t\,dt = \int \frac{\sin t}{\cos t}\,dt = -\int \frac{du}{u} \qquad \cos t = u$$

$$-\sin t\,dt = du$$

$$= -\ln u + c_3$$

$$= -\ln(\cos t) + c_3$$

$$\Rightarrow \int G(t)\,dt = \left(-\frac{1}{2}e^{-t^2}\right)\vec{i} + \left(2\sqrt{\ln t}\right)\vec{j} + \left(-\ln(\cos t)\right)\vec{k} + c$$

4) $\lim\limits_{t\to -2} \dfrac{t^2-4}{t+2} = \lim\limits_{t\to -2} \dfrac{(t-2)\cancel{(t+2)}}{\cancel{t+2}} = \lim\limits_{t\to -2} t-2 = -4$

$\lim\limits_{t\to -2} \ln(t+3) = \ln 1 = 0$

$\lim\limits_{t\to -2} \dfrac{\sin(t+2)}{t+2} = \lim\limits_{u\to 0} \dfrac{\sin u}{u}$ $\quad t+2=u \quad t\to -2 \quad u\to 0$

$= 1$

$\Rightarrow \lim\limits_{t\to -2} \left(\dfrac{t^2-4}{t+2}\vec{i} + \ln(t+3)\vec{j} + \dfrac{\sin(t+2)}{t+2}\vec{k} \right) = -4\vec{i} + \vec{k}$

5) $F.G = e^t \cdot \sin t + (\sqrt{1+t^2})(\ln(1+t) + t\ln t$

$$F\times G = \begin{vmatrix} i & j & k \\ e^t & \sqrt{1+t^2} & \ln t \\ \sin t & \ln(1+t) & t \end{vmatrix}$$

$= \left[t\sqrt{1+t^2} - \ln t(\ln(1+t))\right]\vec{i}$

$+ \left[\ln t \sin t - te^t\right]\vec{j}$

$+ \left(e^t \ln(1+t) - \sin t\sqrt{1+t^2}\right)\vec{k}$